6.1100 Spring 2024 Miniquiz #7

Please submit your answers on Gradescope by April 25th, 2024, 11:59pm.
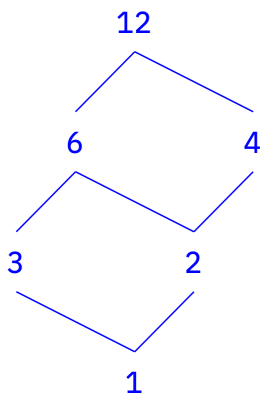
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1. Lattices

Let *L* be the lattice given by the set {1, 2, 3, 4, 6, 12}, and ordered by divisibility. In other words, we say $a \leq b$ as lattice elements if *a* divides *b*.

   a. Draw a **Hasse diagram** for this lattice.

   b. Is this lattice complete? (Circle one:)   **Yes** / No

   c. What is the **top** element in this lattice?   12
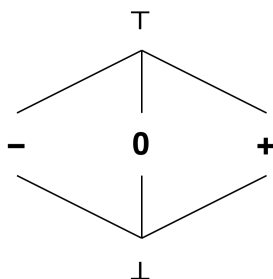
   d. What is the **bottom** element in this lattice?   1

   e. Evaluate the following expressions in this lattice:

Expression	Value (fill in)	Expression	Value (fill in)
$4 \wedge 3 =$	1	$3 \vee (6 \wedge 4) =$	6
$4 \vee 3 =$	12	$3 \wedge (6 \vee 4) =$	3

2. Sign Analysis

This problem is about sign analysis using the following **base lattice**.

Suppose that the analysis tracks the signs of variables **x**, **y**, and **z**, so that the actual lattice used in the analysis contains elements of the form $[x \to s_1, y \to s_2, z \to s_3]$, where s_1, s_2, s_3 are elements of the base lattice.

    a. The **transfer function** of a statement **x = y + z** using this lattice is given by

$$f([x \to s_1, y \to s_2, z \to s_3]) = [x \to s_2 \oplus s_3, y \to s_2, z \to s_3]$$
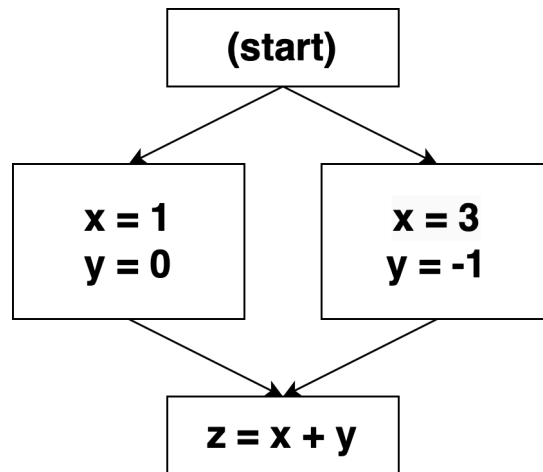
for some binary operation $\oplus$ on the elements of the base lattice. Complete the following table that defines the operation $\oplus$.

$\oplus$	$\bot$	$-$	0	+	$\top$
$\bot$	$\bot$	$\bot$	$\bot$	$\bot$	$\bot$
$-$	$\bot$	$-$	$-$	$\top$	$\top$
0	$\bot$	$-$	**0**	**+**	$\top$
+	$\bot$	$\top$	**+**	**+**	$\top$
$\top$	$\bot$	$\top$	$\top$	$\top$	$\top$

    b. Fill in the blanks below in the transfer function of a statement **y = x**.

$$f([x \to s_1, y \to s_2, z \to s_3]) = [x \to s_1, y \to s_1, z \to s_3]$$

Suppose we are performing sign analysis on the following CFG.

```
          ┌──────────────┐
          │   (start)    │
          └──────────────┘
            ╱          ╲
┌──────────────┐    ┌──────────────┐
│    x = 1     │    │    x = 3     │
│    y = 0     │    │    y = -1    │
└──────────────┘    └──────────────┘
            ╲          ╱
          ┌──────────────┐
          │   z = x + y  │
          └──────────────┘
```

c. According to our sign analysis, what is the lattice point associated to the program point *after* the node **z = x + y**? (Fill in the blanks below.)

$$[x \rightarrow +, y \rightarrow \top, z \rightarrow \top]$$

d. As a human, what is the most precise sign information that you can determine for the program point after the node **z = x + y**? (Fill in the blanks below.)

$$[x \rightarrow +, y \rightarrow \top, z \rightarrow +]$$