# 6.1100 Spring 2024 Miniquiz #7

*Please submit your answers on Gradescope by April 25th, 2024, 11:59pm.*

**Name:**

**Email:**

## 1. Lattices

Let $L$ be the lattice given by the set {1, 2, 3, 4, 6, 12}, and ordered by divisibility. In other words, we say $a \leq b$ as lattice elements if $a$ divides $b$.

a. Draw a **Hasse diagram** for this lattice.

b. Is this lattice complete? (Circle one:) Yes / No

c. What is the **top** element in this lattice? _____

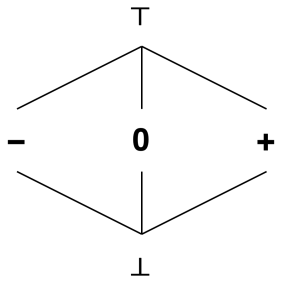
d. What is the **bottom** element in this lattice? _____

e. Evaluate the following expressions in this lattice:

| Expression | Value (fill in) | Expression | Value (fill in) |
|---|---|---|---|
| $4 \wedge 3 =$ | | $3 \vee (6 \wedge 4) =$ | |
| $4 \vee 3 =$ | | $3 \wedge (6 \vee 4) =$ | |

## 2. Sign Analysis

This problem is about sign analysis using the following **base lattice**.




Suppose that the analysis tracks the signs of variables **x**, **y**, and **z**, so that the actual lattice used in the analysis contains elements of the form $[\mathbf{x} \to s_1, \mathbf{y} \to s_2, \mathbf{z} \to s_3]$, where $s_1$, $s_2$, $s_3$ are elements of the base lattice.

a. The **transfer function** of a statement **x = y + z** using this lattice is given by

$$f([\mathbf{x} \to s_1, \mathbf{y} \to s_2, \mathbf{z} \to s_3]) = [\mathbf{x} \to s_2 \oplus s_3, \mathbf{y} \to s_2, \mathbf{z} \to s_3]$$

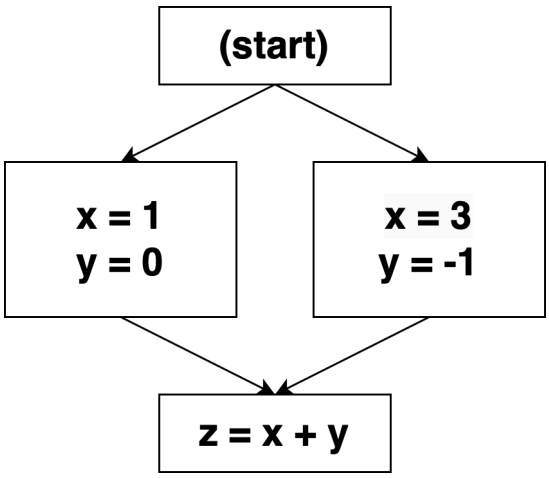
for some binary operation $\oplus$ on the elements of the base lattice. Complete the following table that defines the operation $\oplus$.

| $\oplus$ | $\bot$ | – | 0 | + | $\top$ |
|---|---|---|---|---|---|
| $\bot$ | $\bot$ | $\bot$ | $\bot$ | $\bot$ | $\bot$ |
| – | $\bot$ | | | | $\top$ |
| 0 | $\bot$ | | | | $\top$ |
| + | $\bot$ | | | | $\top$ |
| $\top$ | $\bot$ | $\top$ | $\top$ | $\top$ | $\top$ |

b. Fill in the blanks below in the transfer function of a statement **y = x**.

$f([\mathbf{x} \to s_1, \mathbf{y} \to s_2, \mathbf{z} \to s_3]) = [\mathbf{x} \to$ _____, $\mathbf{y} \to$ _____, $\mathbf{z} \to$ _____$]$

Suppose we are performing sign analysis on the following CFG.




c. According to our sign analysis, what is the lattice point associated to the program point *after* the node **z = x + y**? (Fill in the blanks below.)

[**x** → _____, **y** → _____, **z** → _____]

d. As a human, what is the most precise sign information that you can determine for the program point after the node **z = x + y**? (Fill in the blanks below.)

[**x** → _____, **y** → _____, **z** → _____]